



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

## DISCUSSIONS.

## RELATING TO INDETERMINATE FORMS.

I. A QUESTION WITH RESPECT TO  $0/0$ .

By J. W. NICHOLSON, Louisiana State University.

Has  $0/0$ , (a) in particular cases a definite value, or (b) is it always indeterminate?

A generation ago mathematicians, in general, answered (a) affirmatively and (b) negatively; but in recent years, as a rule, they do just the reverse. This change is due to the ruling out of division by zero, because, as is asserted, it is impossible in cases like  $a/0$  and indeterminate in cases like  $0/0$ .

Without advocating the one contention above the other, I invite a careful consideration of the following argument, hoping that some one will point out the fallacy, if fallacy there be.

The locus of

$$y = \frac{x^2 - a^2}{x - a} \quad (1)$$

consists of the two straight lines<sup>1</sup>

$$y - x - a = 0 \quad (2)$$

and

$$x - a = 0, \quad (3)$$

as shown by clearing of fractions, transposing and factoring.

Now when  $x = a$  in (1),  $y$  has a definite and also an indeterminate value.

For, according to the definitions of loci and their equations, the value of  $y$  when  $x = a$  is the ordinate of the point of intersection of the loci of  $x - a = 0$  and (1); that is, of the line  $x - a = 0 \cdots$  (4) and the two lines (2) and (3).

But the point of intersection of (4) and (2) is  $(a, 2a)$ , and that of (4) and (3) is  $(a, 0/0)$ .

Therefore, when  $x = a$  in (1) we have

$$y = 2a \quad \text{and} \quad y = 0/0.$$

It will be seen that the above process of determining the definite value  $2a$  is the simple operation of finding the intersection of two given lines, and is independent of the idea or doctrine of limits.

By the same method the following more general proposition may be proved:  
When  $x = x'$  the definite value of

$$\frac{f(x) - f(x')}{x - x'} \quad \text{is} \quad f_1(x),$$

in which  $f_1(x)$  is the derivative of  $f(x)$ .

<sup>1</sup> See, for instance, J. W. Young's *Fundamental Concepts of Algebra and Geometry*, p. 214; Granville's *Differential and Integral Calculus*, p. 171; Townsend and Goodenough's *Essentials of Calculus*, p. 21; McMahon and Snyder's *Differential Calculus*, p. 116; and many others.